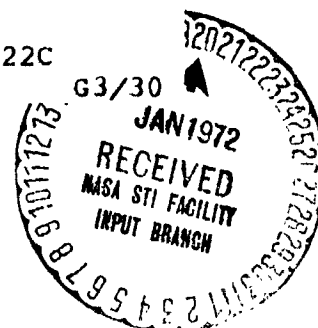




B71 10001

subject Empirical Orbit Determination  
Using Apollo 14 Data -- Case 310

**(CATEGORY)**





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MEMORANDUM FOR FILE

INTRODUCTION

Accurate orbit determination and prediction become especially difficult for the case of a spacecraft in orbit about a body whose gravitational potential field is not well known. Most widely used processing methods employ a model of the gravity field, and hence the results are constrained by the quality of the assumed model. An alternative method is an empirical scheme which models the effects of the gravity field rather than the causes. The Osculating Lunar Elements Program (OLEP) uses such an approach for orbit determination in lunar orbit, representing the state of the satellite as time-varying orbital elements. Estimates obtained for the constant and time-dependent parts of each element are a result of the perturbing effects of the actual gravity field; no model of the field is assumed.

In this study the orbit determination and prediction capabilities of OLEP are investigated using Command Service Module (CSM) Doppler tracking data from the lunar parking orbit of Apollo 14. Long and short arc solutions are presented, and the behavior of the estimated osculating orbital elements is studied. Correspondence between residuals and topographic features is shown.

MATHEMATICAL DESCRIPTION

The OLEP\* approach uses time-varying functions for the low-eccentricity orbital elements,  $a$ ,  $e_s = e \sin \omega$ ,  $e_c = e \cos \omega$ ,  $I$ ,  $\Omega$ , and  $m = \omega + M$ , to model the motion of a spacecraft in an

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\*Bullock, M. V. and Ferrari, A. J., "Orbit Determination For Lunar Parking Orbits Using Time-Varying Orbital Elements," NASA Contractor Report 110008, May 1970.



Apollo-type lunar orbit. A typical element is represented as

$$\Omega(t) = \Omega_0 + \Omega_1 t$$

with quadratic terms included for  $e_s$ ,  $e_c$ , and  $m$  in the case of regressions of more than two passes of data. The numerical singularities associated with nearly equatorial orbits are avoided by defining these low-eccentricity elements in a special selenocentric frame which represents any orbit as a polar orbit. This transformation is accomplished by rotating the initial estimate of the selenocentric state at epoch through two of its associated Euler angles  $(\hat{\Omega}, \hat{I})$ .

The semi-major axis does not appear as an explicit solution parameter. The constant and time-dependent portions of the remaining five elements constitute the parameter set for which estimates are obtained during the orbit determination process. The estimate for  $m_1$ , the linear portion of the modified anomaly, is used to imply a corresponding semi-major axis by using the classical Kepler relationship

$$a = \left[ \frac{\mu}{m_1^2} \right]^{1/3},$$

where  $\mu$  is the Newtonian constant times the lunar mass.

#### DATA ANALYSIS

Orbit determinations are performed using free flight and minimally corrupted data from Apollo 14. Since propulsive maneuvers are not modeled in OLEP, the best orbit determination results are to be expected when free flight data are processed. Data acquired during coupled attitude maneuvers, such as land-mark tracking, should be of free flight quality, but in practice the jets are never perfectly balanced so a slight translation is imparted to the spacecraft. At least two passes of data are processed in each case, since one pass does not contain enough orbital period and time-varying information to enable OLEP to predict accurately. The effects of the CSM separation burn are also shown.



a. Processing of Free Flight Data

Residuals resulting from two-pass regressions of pre-DOI tracking data are shown in Figures 1 and 2. This orbit is characterized by a perilune of 8n.mi. and an apolune of 60n.mi. above the lunar surface. The two-pass fit, two-pass predict residuals for the four sets of data are virtually identical, with peak-to-peak values of  $\pm 1.6$  feet/second. The excellent quality of the solutions is apparent in the fact that the prediction residuals exhibit almost no secular growth and that they maintain the characteristic short-periodic shape of the fit residuals. As expected, some growth occurs when a two-pass solution is predicted for five passes (Figure 2), but the solution still describes the orbit very well for the entire time span. Peak-to-peak growth in the prediction region is less than double that of the fit region, and the residuals still display some short-periodic shape.

High quality results are also obtained when the OLEP process is extended to multi-pass regressions. Two solutions were obtained, one from the seven pre-DOI passes discussed above and one from six post-DOI passes; the residuals from both are shown in Figure 3. The post-DOI orbit was near-circular with both perilune and apolune about 60n.mi. above the lunar surface. In peak-to-peak value and residual shape these results are extremely similar to the two-pass fits.

b. Processing of Corrupted Data

Figure 4 shows several examples of two-pass processing of pre-DOI data acquired during various indicated maneuvers. The solution from passes 3 and 4 was somewhat corrupted by landmark tracking between passes 3 and 4 even though the maneuver was coupled, and the prediction residuals are markedly different from those which resulted from free flight data processing. The peak-to-peak value is more than double that in the fit region, and the characteristic shape is lost. Similar results occur when a solution from free flight data is predicted through data containing maneuvers. LM RCS jet hot firing tests were performed during pass 11, and the undocking and separation maneuvers occurred in pass 12. These events changed the orbit so that the solutions from the previous passes no longer describe it adequately.

OSCULATING ELEMENT COMPARISONS

A further indication of the quality of the solutions obtained by the OLEP method is consistency in the behavior of the estimated parameters. Local estimates of the orbital elements



are obtained from two-pass fits using passes 4-10, and a single fit over the entire data span gives a set of long-term estimates. The long-arc fit should provide more accurate estimates of the elements than the short-arc fits because the longer interval gives the process more information about the time behavior of the elements.

Figures 5 and 6 show comparisons of eccentricity, inclination, longitude of the ascending node, and argument of perilune from short and long-arc fits. The perturbing effects of the earth and the sun on the elements were analytically removed from the long-arc solution. The resulting variations in the elements are presented to show the effects of non-central gravitational features on the spacecraft. Extremely accurate estimates of  $e$ ,  $\Omega$ , and  $\omega$  were obtained from the short fits. The local estimates follow the slopes of the long-arc estimates very closely. The inclination estimates are not as consistent, but this is not an unexpected result since the inclination is the most difficult parameter to determine.

#### RESIDUAL ANALYSIS

In Figure 7 the residuals from passes 4-10 are given as a function of the longitude of the sub-vehicle point, with the consecutive passes being overlaid. The OLEP process estimates only the secular and long-period perturbing effects of the gravity field, hence the residuals are short-periodic line-of-sight velocity errors. The consistency of the residual pattern from pass to pass is especially striking. Prominent topographic features over which the spacecraft passed are noted on the figure. Their effects on the orbit can be seen in the correspondence between these features and the occurrence of the largest residuals.

#### SUMMARY AND CONCLUSIONS

It has been demonstrated that highly accurate orbit determination and prediction can be performed with no assumptions about the lunar gravity field using the OLEP process. The residuals from the processing of free flight Apollo 14 data experience minimal secular growth and exhibit a characteristic short-periodic shape. The orbital elements resulting from two pass solutions behave in a consistent manner when compared with elements from a long-arc solution. The quality of the results is reduced using data acquired during maneuvers; even coupled maneuvers impart some change to the orbit since the jets are



not perfectly balanced during these firings. Consistency in the residual pattern from consecutive passes of free flight data is shown, and the residuals are correlated with sub-vehicle topographic features.

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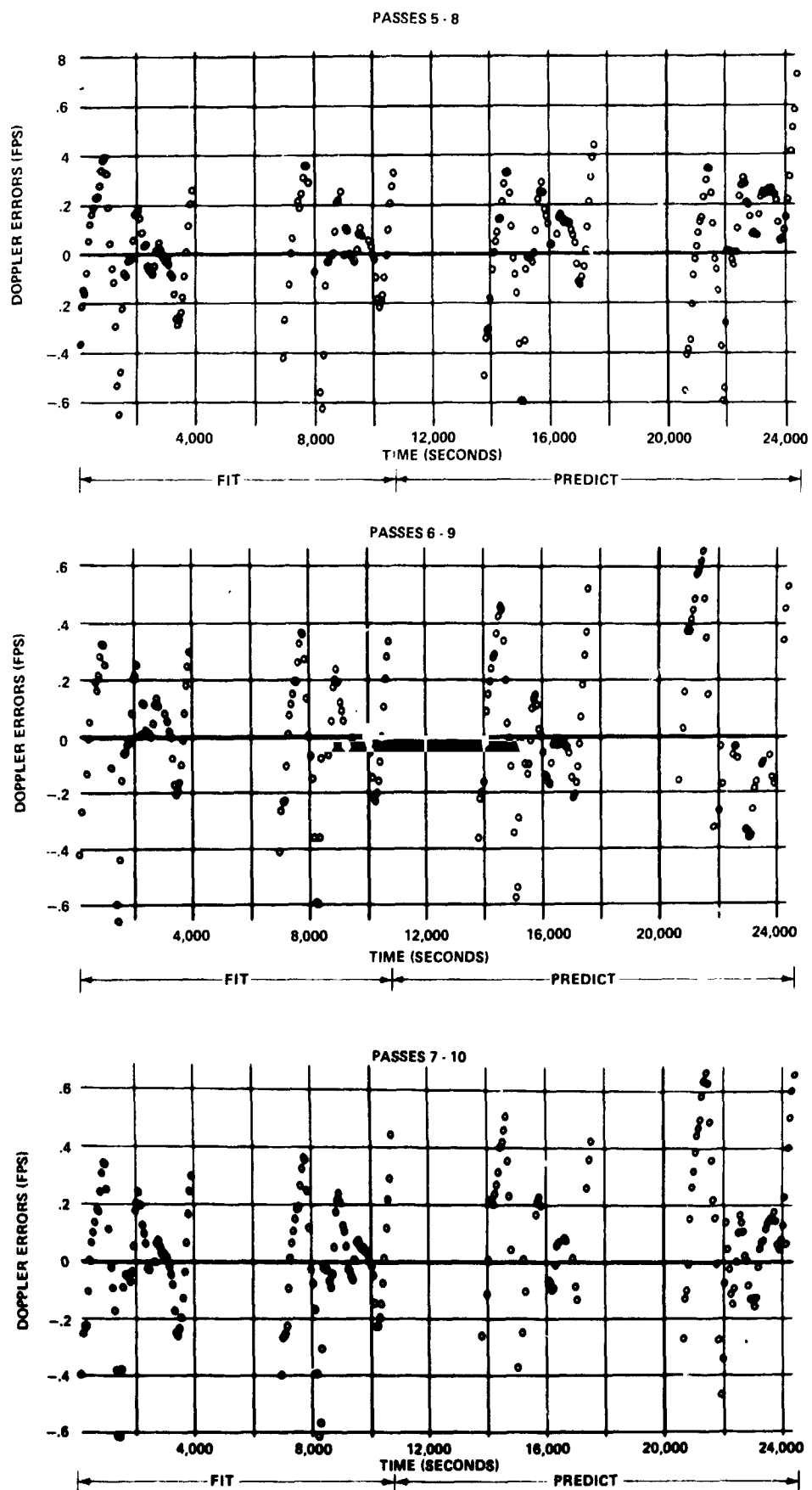


FIGURE 1 - TWO-PASS FITS OF FREE FLIGHT DATA

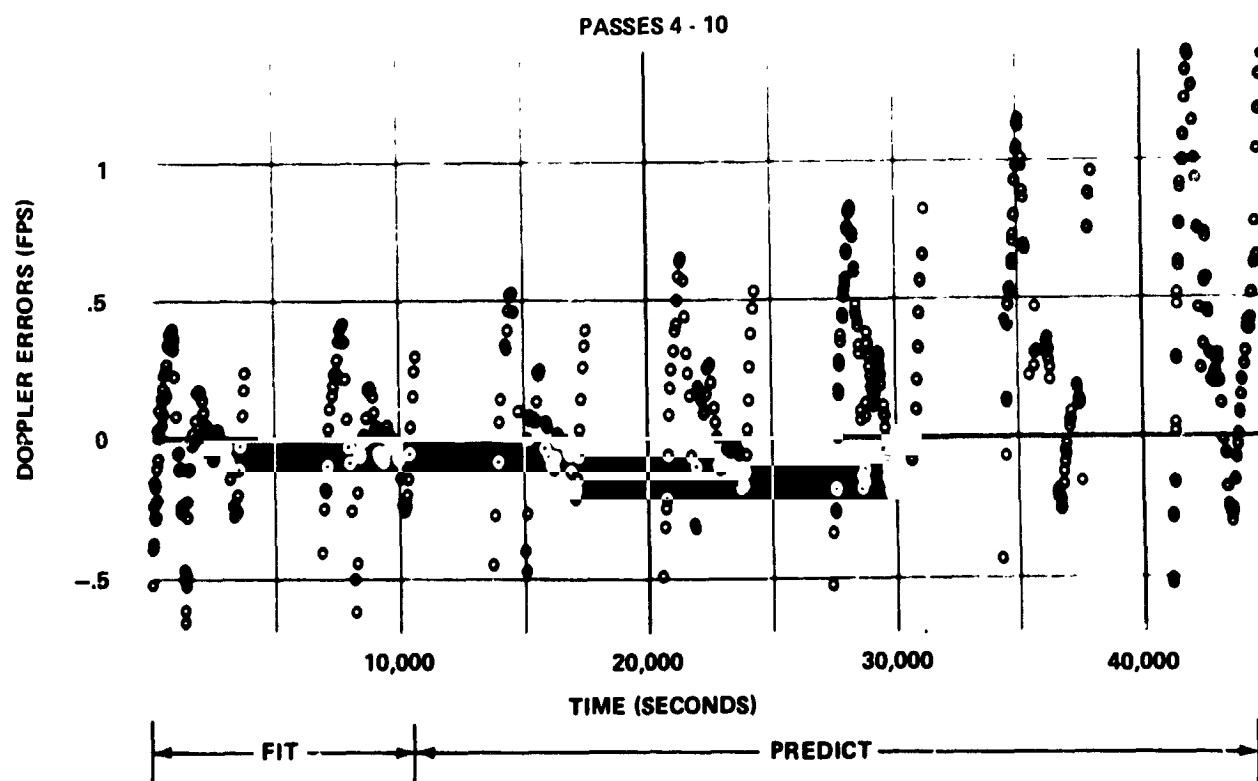
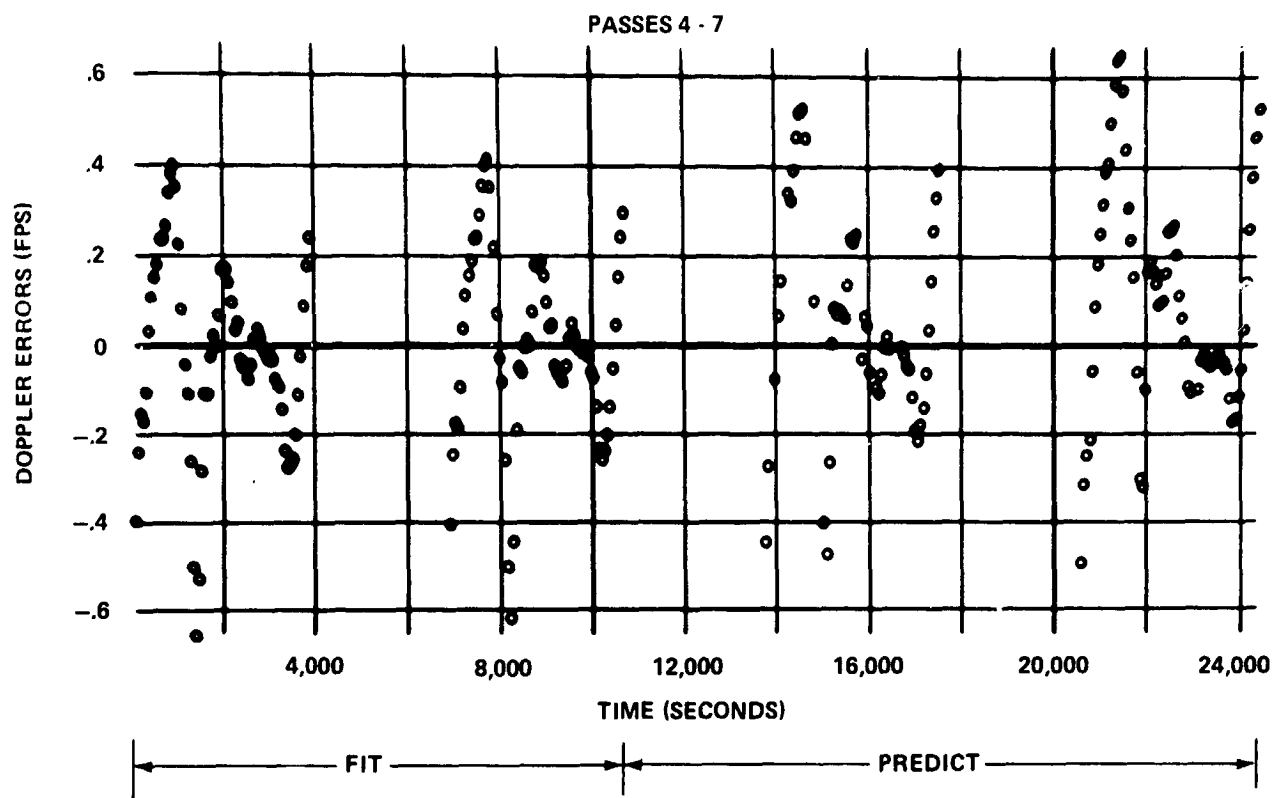


FIGURE 2 - PROCESSING OF PASSES 4 AND 5



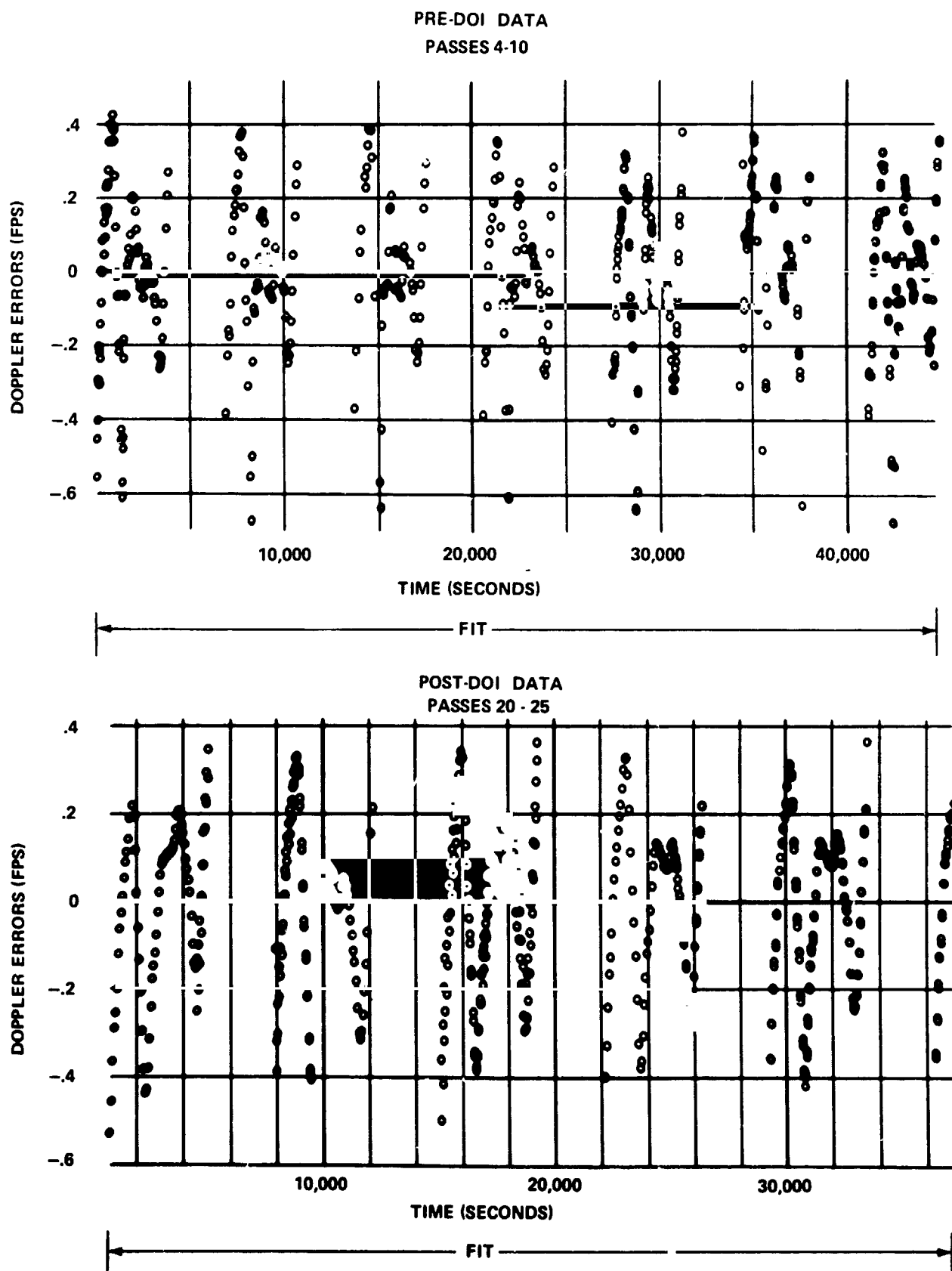


FIGURE 3 - MULTI-PASS PROCESSING OF FREE FLIGHT DATA

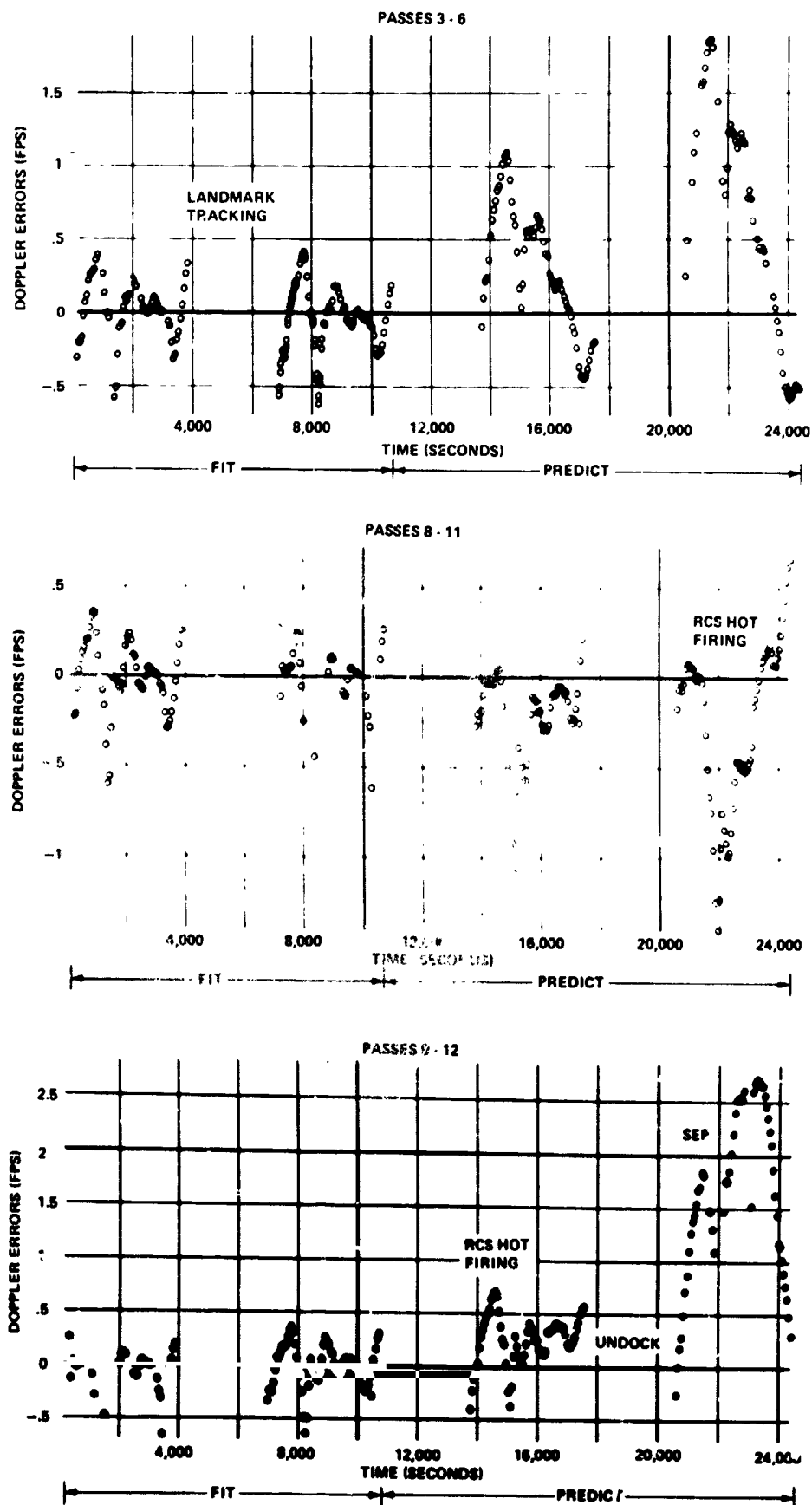
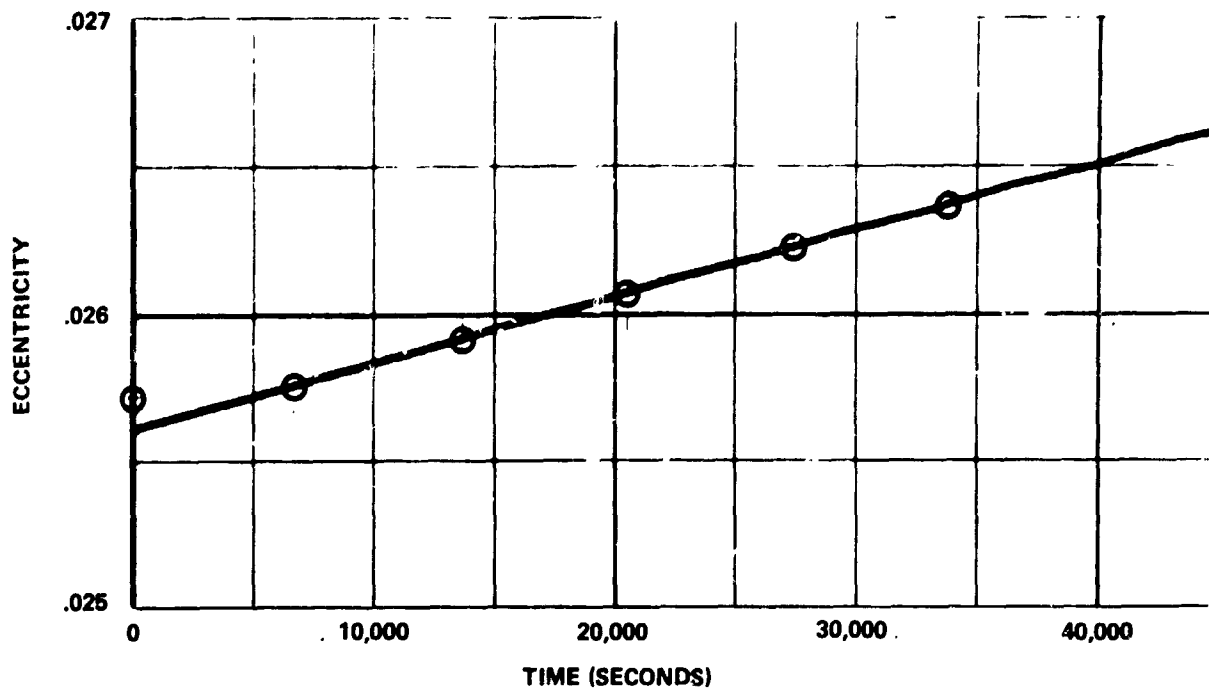
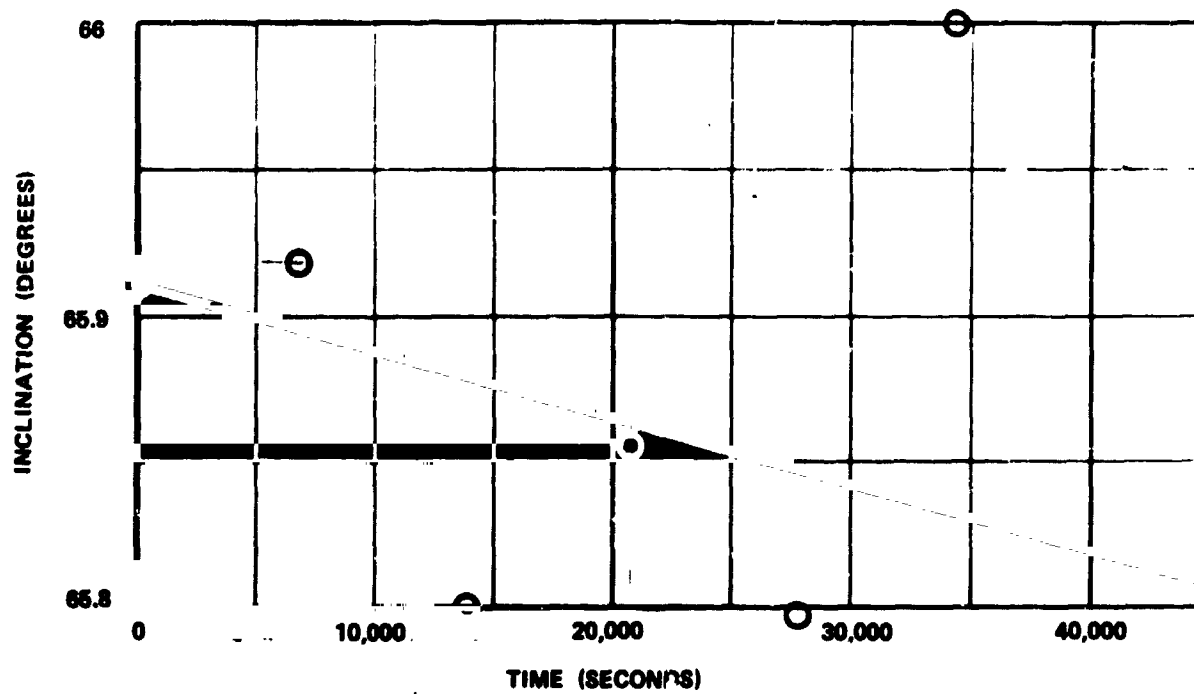


FIGURE 4 - PROCESSING OF CORRUPTED DATA

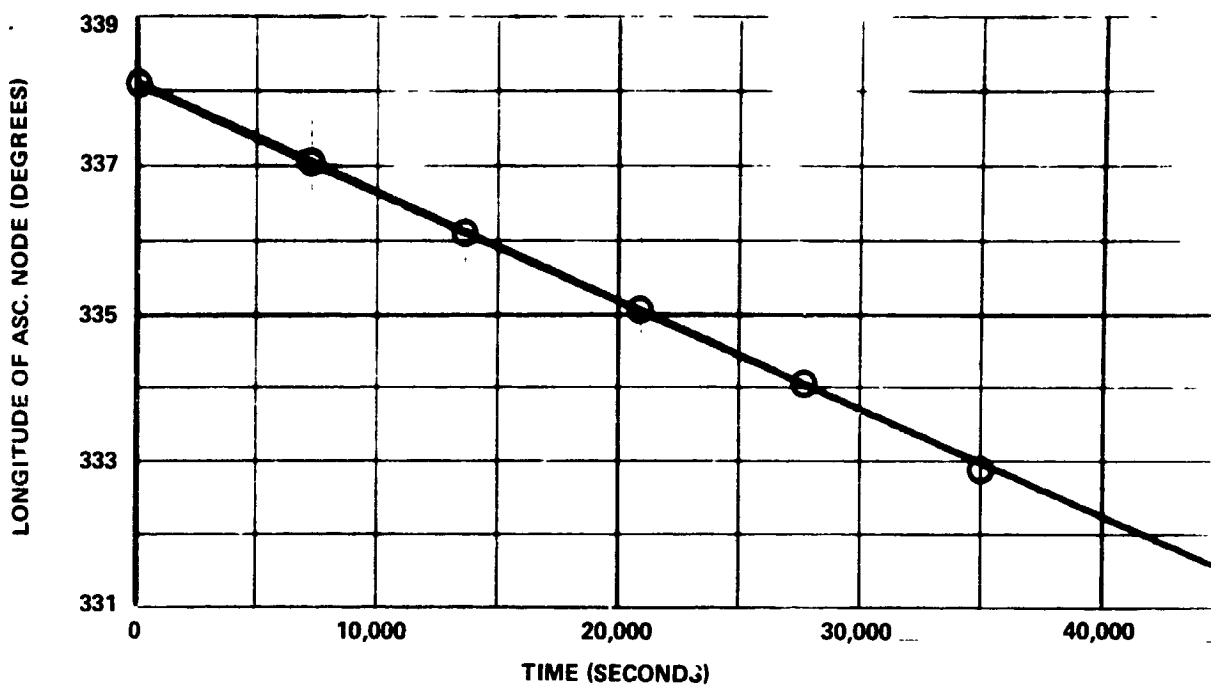


ECCENTRICITY AS A  
FUNCTION OF TIME

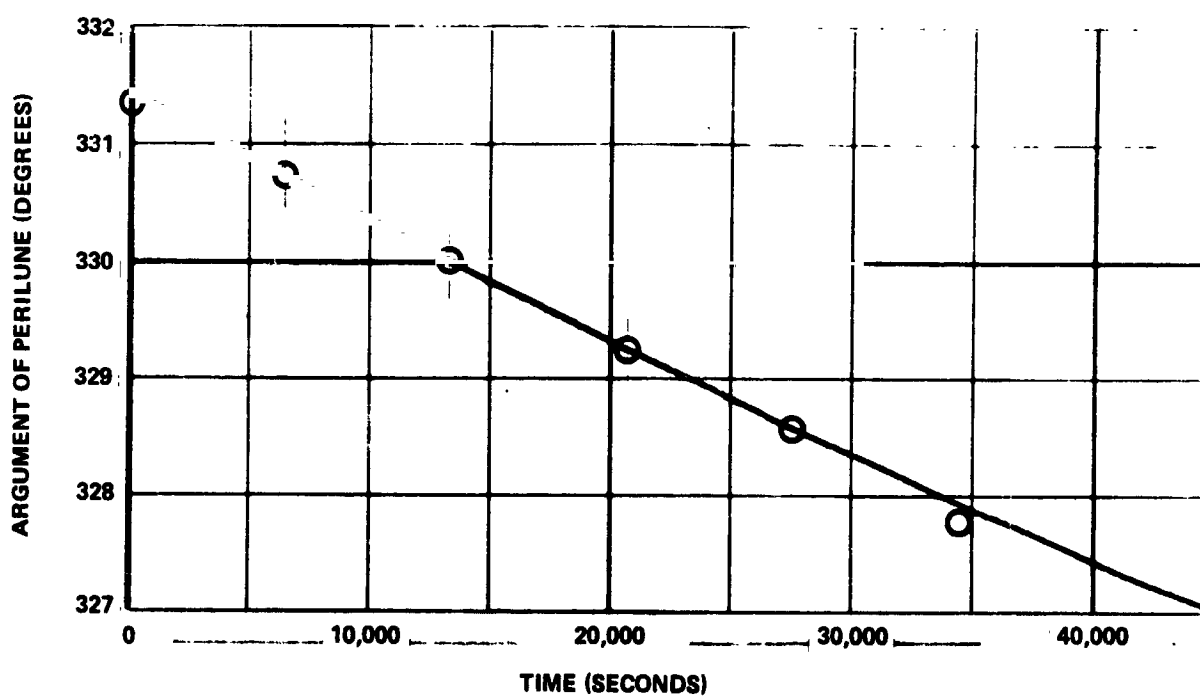


INCLINATION AS A  
FUNCTION OF TIME

FIGURE 5 - APOLLO 14 ORBITAL ELEMENTS, PASSES 4 - 10



ASCENDING NODE AS  
A FUNCTION OF TIME



ARGUMENT OF PERILUNE  
AS A FUNCTION OF TIME

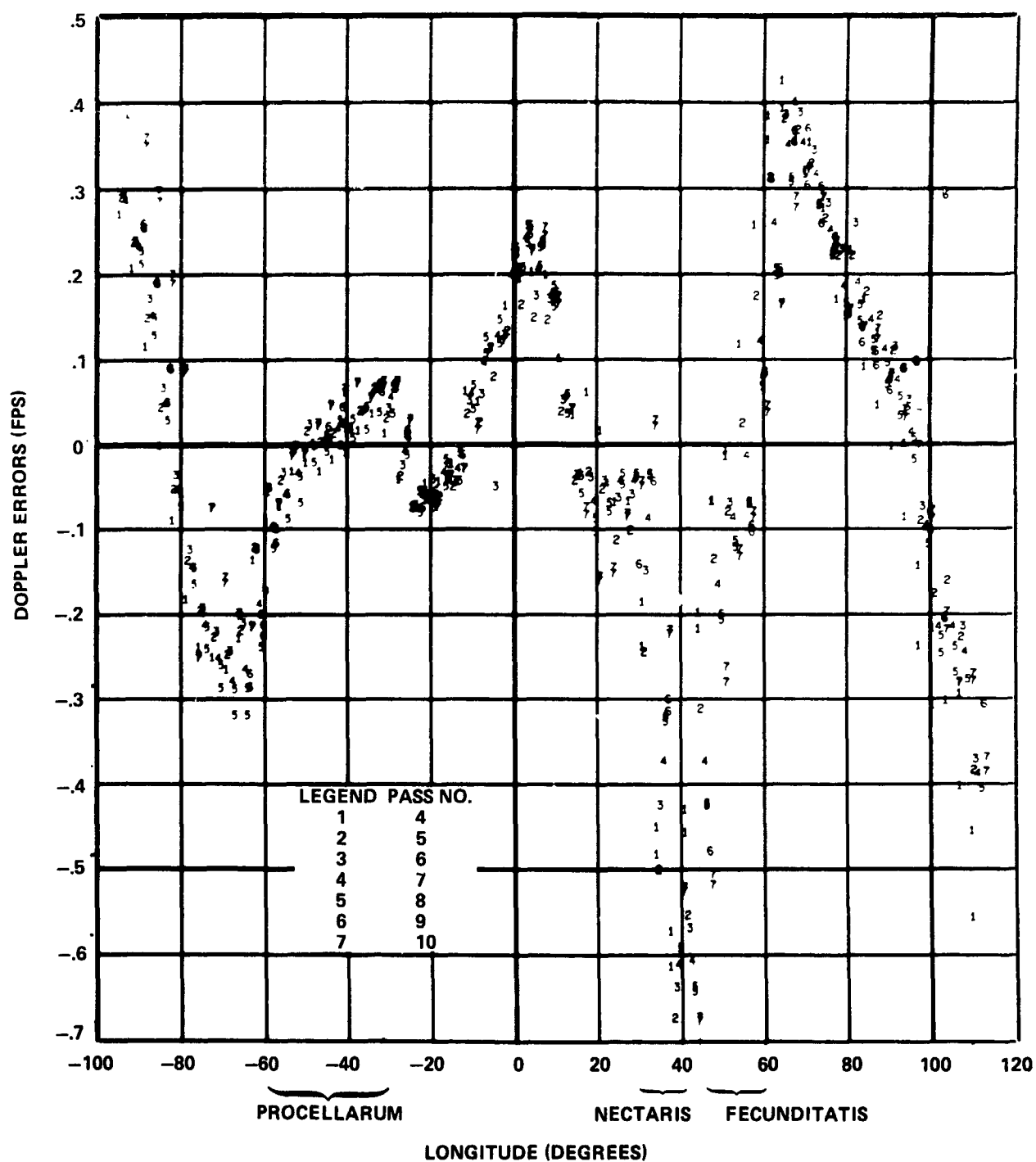


FIGURE 7 - APOLLO 14 RESIDUALS AS A FUNCTION OF LONGITUDE, PASSES 4 - 10